

Double neutron-proton differential transverse flow as a probe for the high-density behavior of the nuclear symmetry energy

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Abstract

The double neutron-proton differential transverse flow taken from two reaction systems using different isotopes of the same element is studied at incident beam energies of 400 and 800 MeV/nucleon within the framework of an isospin- and momentum-dependent hadronic transport model IBUU04. The double differential flow is found to retain about the same sensitivity to the density dependence of the nuclear symmetry energy as the single differential flow in the more neutron-rich reaction. Because the double differential flow reduces significantly both the systematic errors and the influence of the Coulomb force, it is thus more effective probe for the high-density behavior of the nuclear symmetry energy.

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I. INTRODUCTION

The density dependence of the nuclear symmetry energy is not only important for nuclear physics, but also crucial for many astrophysical processes, such as the structure of neutron stars and the dynamical evolution of proto-neutron stars [1, 2]. Heavy-ion reactions induced by neutron-rich nuclei, especially radioactive beams, provide a unique opportunity to constrain the equation of state (EOS) of asymmetric nuclear matter [3, 4, 5]. Though considerable progress has been made recently in determining the density dependence of the nuclear symmetry energy around the normal nuclear matter density from studying the isospin diffusion in heavy-ion reactions at intermediate energies [6, 7, 8], much more work is still needed to probe the high-density behavior of the nuclear symmetry energy.

A key task is to identify experimental observables sensitive to the density dependence of the nuclear symmetry energy, especially at high densities. Several potentially useful observables, such as, the free neutron/proton ratio [9], the isospin fractionation [10, 11, 12, 13, 14, 15], the neutron-proton correlation function [16], $t/{}^3\text{He}$ [17, 18], the isospin diffusion [19, 20], the proton differential elliptic flow [21] and the π^-/π^+ ratio [22, 23, 24, 25] have been proposed in the literature. The concept of the neutron-proton differential flow was first introduced by one of us [26] several years ago. It was argued that the neutron-proton differential flow minimizes influences of the isoscalar potential but maximizes effects of the symmetry potential. It can also reduce effects of other dynamical ingredients in intermediate energy heavy-ion reactions. It is therefore among the most promising probes for the high density behavior of the nuclear symmetry energy.

In order to extract accurately information about the symmetry energy one has to reduce as much as possible the systematic errors involved in the experimental observables. Moreover, the long range Coulomb force on charged particles may play an important role in these observables. If at all possible, one would like to disentangle effects of the symmetry energy from those due to the Coulomb force. Very often, this is impossible. One would thus like to construct observables that can reduce the Coulomb effects as much as possible. Ratios and/or differences of two observables from a pair of reactions using different isotopes of the same element are among the promising candidates to reduce both the systematic errors and the Coulomb effects. Whether to use the ratio or the difference to construct the desired observable depends on the nature of the observables involved. For the neutron/proton ratio

of pre-equilibrium nucleons and the π^-/π^+ ratio, for instance, it is natural to construct their double ratios as was recently done in Refs.[27, 28, 29]. However, the neutron-proton differential flow is additive, it is more useful to construct the double differences instead of ratios. In the present work, we investigate the double neutron-proton differential transverse flow from the two reactions of $^{132}\text{Sn}+^{124}\text{Sn}$ and $^{112}\text{Sn}+^{112}\text{Sn}$ at beam energies of 400 and 800 MeV/nucleon. It is found to have the same sensitivity to the density dependence of the nuclear symmetry energy as the single neutron-proton differential flow in the neutron-rich reaction system $^{132}\text{Sn}+^{124}\text{Sn}$. Besides having smaller systematic errors, the double differential flow is shown indeed to reduce significantly the Coulomb effects.

II. A BRIEF INTRODUCTION TO THE IBUU04 TRANSPORT MODEL

Our present studies are based on the transport model IBUU04, in which nucleons, Δ and N^* resonances as well as pions and their isospin-dependent dynamics are included. The initial neutron and proton density distributions of the projectile and target are obtained by using the relativistic mean field theory. We use the isospin-dependent in-medium nucleon-nucleon (NN) elastic cross sections from the scaling model according to nucleon effective masses [8]. For the inelastic cross sections we use the experimental data from free space NN collisions since at higher incident beam energies the NN cross sections have no evident effects on the slope of neutron-proton differential flow [8]. The total and differential cross sections for all other particles are taken either from experimental data or obtained by using the detailed balance formula. The isospin dependent phase-space distribution functions of the particles involved are solved by using the test-particle method numerically. The isospin-dependence of Pauli blockings for fermions is also considered. Details can be found in Refs. [8, 9, 28, 30, 31, 32]. The momentum- and isospin-dependent single nucleon potential (MDI) adopted [31] is

$$\begin{aligned}
U(\rho, \delta, \mathbf{p}, \tau) = & A_u(x) \frac{\rho_{\tau'}}{\rho_0} + A_l(x) \frac{\rho_{\tau}}{\rho_0} \\
& + B \left(\frac{\rho}{\rho_0} \right)^{\sigma} (1 - x\delta^2) - 8x\tau \frac{B}{\sigma + 1} \frac{\rho^{\sigma-1}}{\rho_0^{\sigma}} \delta \rho_{\tau'} \\
& + \frac{2C_{\tau, \tau}}{\rho_0} \int d^3\mathbf{p}' \frac{f_{\tau}(\mathbf{r}, \mathbf{p}')}{1 + (\mathbf{p} - \mathbf{p}')^2/\Lambda^2} \\
& + \frac{2C_{\tau, \tau'}}{\rho_0} \int d^3\mathbf{p}' \frac{f_{\tau'}(\mathbf{r}, \mathbf{p}')}{1 + (\mathbf{p} - \mathbf{p}')^2/\Lambda^2}.
\end{aligned} \tag{1}$$

The detailed values of the parameters can be found in Ref. [7, 8, 31]. With the above single particle potential $U(\rho, \delta, \mathbf{p}, \tau)$, for a given value x , one can readily calculate the symmetry energy $E_{\text{sym}}(\rho)$ as a function of density. Noticing that the isospin diffusion data from NSCL/MSU have constrained the value of x to be between 0 and -1 for nuclear matter densities less than about $1.2\rho_0$ [7, 8], in the present work, as an example, we also consider the two values of $x = 0$ and $x = -1$. Shown in Fig. 1 is the density dependence of the nuclear symmetry energy with the two x values. It is seen that the case of $x = 0$ gives a softer symmetry energy than that of $x = -1$ and the difference becomes larger at higher densities.

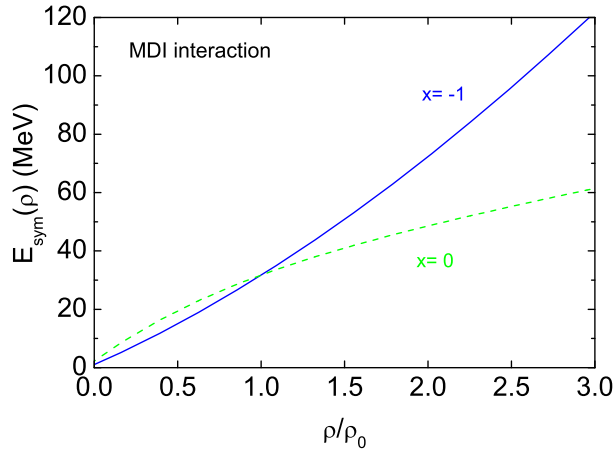


FIG. 1: (Color online) Density dependence of nuclear symmetry energy using the MDI interaction with $x = 0$ and $x = -1$.

III. RESULTS AND DISCUSSIONS

To study the high-density behavior of the symmetry energy, it is useful to know the maximal baryon density reached in a given reaction. The maximal density depends not only on the incident beam energy, but also on the impact parameter as well as the reaction system. Fig. 2 shows the time evolution of the average baryon density in the central cell reached in $^{112}\text{Sn}+^{112}\text{Sn}$ reaction at beam energies of 400 and 800 MeV/nucleon with an impact parameter of $b = 5$ fm. One can see that for 400 MeV/nucleon, the maximal baryon density reached is about $1.9\rho_0$ while it is about $2.4\rho_0$ for 800 MeV/nucleon. One can also

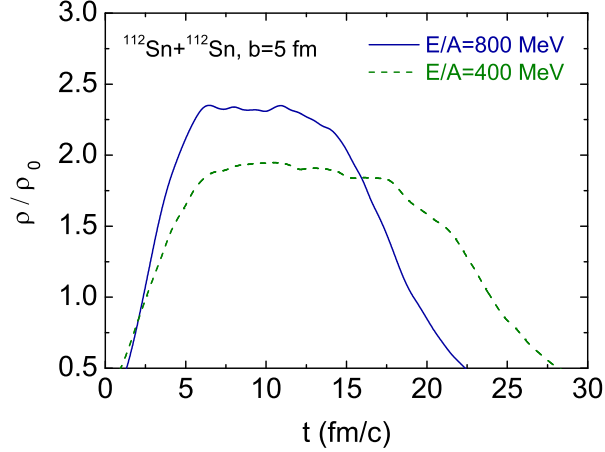


FIG. 2: (Color online) Time evolution of the average baryon density in the central cell reached in the $^{112}\text{Sn}+^{112}\text{Sn}$ reaction at the incident beam energies of 400 and 800 MeV/nucleon with an impact parameter of $b = 5$ fm.

see that at the higher incident energy, the life time of the high density nuclear matter is shorter as expected.

The neutron-proton differential transverse flow was defined as [22, 26]

$$\begin{aligned}
 F_{n-p}^x(y) &\equiv \frac{1}{N(y)} \sum_{i=1}^{N(y)} p_i^x(y) w_i \\
 &= \frac{N_n(y)}{N(y)} \langle p_n^x(y) \rangle - \frac{N_p(y)}{N(y)} \langle p_p^x(y) \rangle
 \end{aligned} \tag{2}$$

where $N(y)$, $N_n(y)$ and $N_p(y)$ are the number of free nucleons, neutrons and protons, respectively, at rapidity y ; $p_i^x(y)$ is the transverse momentum of the free nucleon at rapidity y ; $w_i = 1$ (-1) for neutrons (protons); and $\langle p_n^x(y) \rangle$ and $\langle p_p^x(y) \rangle$ are respectively the average transverse momenta of neutrons and protons at rapidity y . It is seen from Eq. (2) that the constructed neutron-proton differential transverse flow depends not only on the proton and neutron transverse momenta but also on their relative multiplicities. We stress that the neutron-proton differential flow combines effects due to both the isospin fractionation and the different transverse flows of neutrons and protons. It is noticed that the neutron-proton differential transverse flow is not simply the difference of the neutron and proton transverse flows. Instead, it depends also on the isospin fractionation at the rapidity y . To see this point more clearly, let's consider two special cases. If neutrons and protons have the same average transverse momentum in the reaction plane but different multiplicities in each ra-

pidity bin, i.e., $\langle p_n^x(y) \rangle = \langle p_p^x(y) \rangle = \langle p^x(y) \rangle$, and $N_n(y) \neq N_p(y)$, then Eq. (2) is reduced to

$$F_{n-p}^x(y) = \frac{N_n(y) - N_p(y)}{N(y)} \langle p^x(y) \rangle = \delta(y) \cdot \langle p^x(y) \rangle, \quad (3)$$

reflecting effects of the isospin fractionation. On the other hand, if neutrons and protons have the same multiplicity but different average transverse momenta, i.e., $N_n(y) = N_p(y)$ but $\langle p_n^x(y) \rangle \neq \langle p_p^x(y) \rangle$, then Eq. (2) is reduced to

$$F_{n-p}^x(y) = \frac{1}{2}(\langle p_n^x(y) \rangle - \langle p_p^x(y) \rangle). \quad (4)$$

In this case it reflects directly the difference of the neutron and proton transverse flows. In heavy-ion collisions at higher energies [27, 28], generally, for free nucleons in a given rapidity bin, one expects that a stiffer symmetry potential leads to a higher isospin fractionation and also contributes more positively to the transverse momenta of neutrons compared to protons. The neutron-proton differential flow thus combines constructively effects of the symmetry potentials for neutrons and protons.

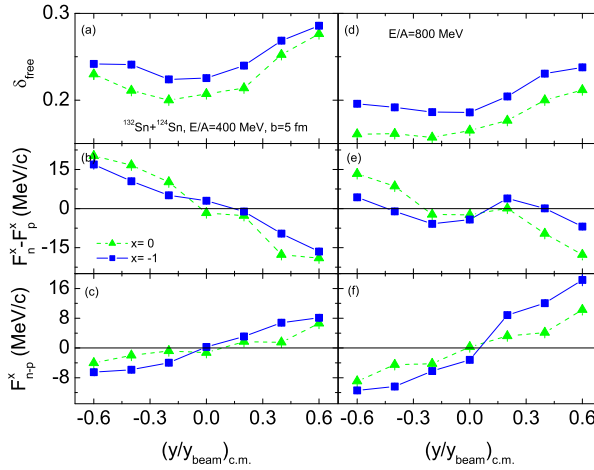


FIG. 3: (Color online) Rapidity distribution of the isospin asymmetry of free nucleons (upper panels), the difference of the average nucleon transverse flows (middle panels) and the neutron-proton differential transverse flow (lower panels) from $^{132}\text{Sn}+^{124}\text{Sn}$ reaction at the incident beam energies of 400, 800 MeV/nucleon and $b = 5$ fm with two symmetry energies of $x = 0$ and $x = -1$.

Shown in Fig. 3 are the rapidity distribution of the isospin asymmetry of free nucleons (upper panels), the difference of the average nucleon transverse flows (middle panels) and

the neutron-proton differential transverse flow (lower panels) from the $^{132}\text{Sn}+^{124}\text{Sn}$ reaction at incident beam energies of 400, 800 MeV/nucleon and an impact parameter of $b = 5$ fm with the two symmetry energies of $x = 0$ and $x = -1$. It is seen from the upper panels of Fig. 3 that a larger isospin asymmetry of free nucleons (stronger isospin fractionation) is obtained for the stiffer symmetry energy ($x = -1$). It is interesting to see from the bottom panels of Fig. 3 that the stiffer symmetry energy ($x = -1$) leads to clearly a stronger neutron-proton differential transverse flow than the softer symmetry energy ($x = 0$). From the middle panels we notice that the difference of the average nucleon flows exhibits less sensitivity to the symmetry energy compared with the neutron-proton differential transverse flow. Normally, the Coulomb potential dominates over the symmetry potential for protons, consequently protons have higher average transverse momenta than neutrons, leading to the negative (positive) values of the $F_n^x - F_p^x$ at forward (backward) rapidities.

We examine the beam energy dependence of the neutron-proton differential transverse flow in the lowest two panels (c) and (f) of Fig. 3. As one expects, with the same symmetry energy, the slope of the neutron-proton differential transverse flow around the mid-rapidity is larger for the higher incident beam energy. This is mainly because a denser nuclear matter is formed at higher incident beam energy (shown in Fig. 2). It then leads to a stronger symmetry potential and thus higher transverse momenta for neutrons compared to protons. The magnitude of the neutron-proton differential transverse flow at 800 MeV/nucleon is much larger than that at 400 MeV/nucleon and it is thus easier to be measured experimentally although the net effect of the symmetry potential on the neutron-proton differential transverse flow is not much larger than that at 400 MeV/nucleon.

In order to reduce the systematic errors, one can study the relative values of some observables from two similar reaction systems. In the present work, we thus also studied the less neutron-rich reaction system $^{112}\text{Sn}+^{112}\text{Sn}$ with the same reaction conditions as a reference. Fig. 4 shows the rapidity distribution of the neutron-proton differential transverse flow in the semi-central reaction of $^{112}\text{Sn}+^{112}\text{Sn}$ at the same incident beam energies of 400 and 800 MeV/nucleon. Comparing with the case of $^{132}\text{Sn}+^{124}\text{Sn}$, we can see that the slope of the neutron-proton differential transverse flow around mid-rapidity and effects of the symmetry energy become much smaller due to the smaller isospin asymmetry in the reaction of $^{112}\text{Sn}+^{112}\text{Sn}$.

How should one use the reaction of $^{112}\text{Sn}+^{112}\text{Sn}$ as a reference? Since for $^{112}\text{Sn}+^{112}\text{Sn}$

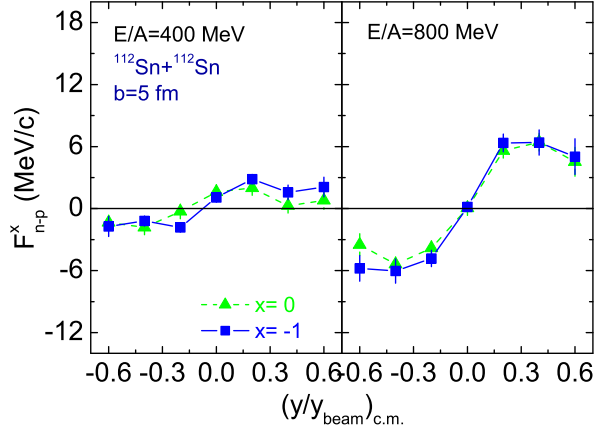


FIG. 4: (Color online) Same as the lowest two panels (c) and (f) of Fig. 3 but for the reaction system of $^{112}\text{Sn}+^{112}\text{Sn}$.

effects of the symmetry energy on neutron-proton differential transverse flow almost disappear, it is the easiest to study the difference of the neutron-proton differential flows (we dub it the double neutron-proton differential flow) from the two reaction systems of $^{132}\text{Sn}+^{124}\text{Sn}$ and $^{112}\text{Sn}+^{112}\text{Sn}$. Fig. 5 shows the rapidity distribution of the double neutron-proton differential transverse flow in the semi-central reactions of Sn+Sn isotopes. At both incident beam energies of 400 and 800 MeV/nucleon, it is interesting to see that the double neutron-proton differential transverse flow around mid-rapidity is essentially zero for the soft symmetry energy of $x = 0$. However, it displays a clear slope with respect to the rapidity for the stiffer symmetry energy of $x = -1$. Moreover, the double neutron-proton differential transverse flow at the higher incident energy indeed exhibits a stronger symmetry energy effect as expected. Furthermore, it is seen that the double neutron-proton differential transverse flow retains about the same symmetry energy effect as the $^{132}\text{Sn}+^{124}\text{Sn}$ reaction. As discussed in Ref. [33], observables coming from many reaction combinations under identical experimental conditions are insensitive to systematic uncertainties due to the apparatus. Theoretically, in transport model calculations, the systematic errors are mostly related to the physical uncertainties of in-medium NN cross sections, techniques of treating collisions, sizes of the lattices in calculating the phase space distributions, techniques in handling the Pauli blocking, etc.. Since the double neutron-proton differential flow is a relative observable from the two similar reaction systems, systematic errors are thus expected to be reduced.

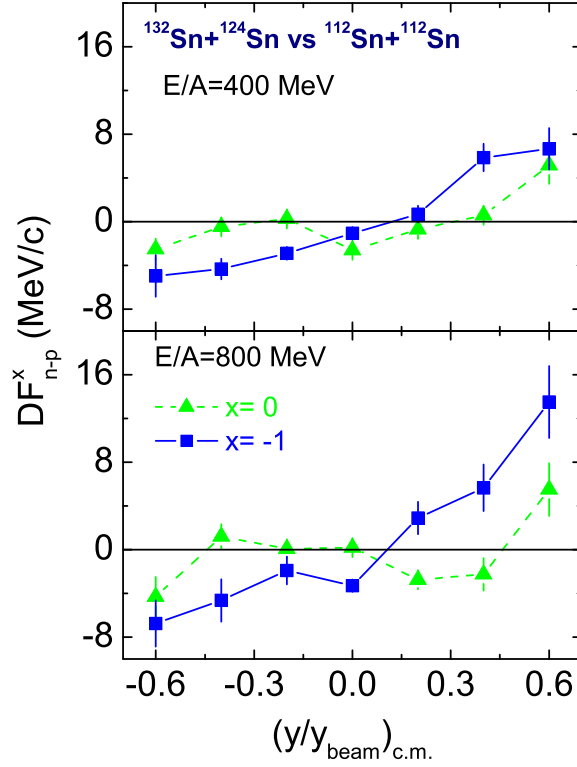


FIG. 5: (Color online) Rapidity distribution of the double neutron-proton differential transverse flow in the semi-central reactions of Sn+Sn isotopes at the incident beam energies of 400 and 800 MeV/nucleon with two symmetry energies of $x = 0$ and $x = -1$.

Moreover, the Coulomb force on charged particles may play important roles. It sometimes competes strongly with the symmetry potentials. One thus has to disentangle carefully effects of the symmetry potentials from those due to the Coulomb potentials. Because the double neutron-proton differential transverse flow is a relative observable, Coulomb effects are also expected to be much reduced. To verify this expectation, we examine in Fig. 6 Coulomb effects on the neutron-proton differential transverse flow (upper two panels) and the double neutron-proton differential transverse flow (lowest panel) in the semi-central reactions of Sn+Sn isotopes at the incident beam energy of 400 MeV/nucleon with the symmetry energy of $x = 0$. From the upper two panels of Fig. 6, one can see that the Coulomb effects reduce the strength of the neutron-proton differential transverse flow. With the Coulomb force, more protons are unbound and have large transverse momenta in the reaction-plane. According to Eq. (2), the strength of the neutron-proton differential transverse flow will

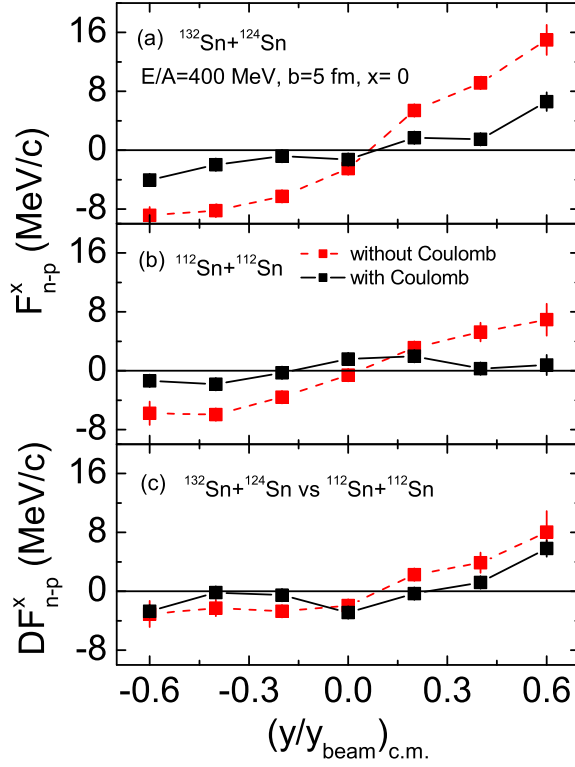


FIG. 6: (Color online) Coulomb effects on the neutron-proton differential transverse flow (upper two panels) and the double neutron-proton differential transverse flow (lowest panel) in the semi-central reactions of Sn+Sn isotopes at the incident beam energy of 400 MeV/nucleon with the symmetry energy of $x = 0$.

be reduced. The lowest panel of Fig. 6 shows that the double neutron-proton differential transverse flow can reduce the effect of long range Coulomb force largely.

IV. SUMMARY

In summary, based on the isospin- and momentum-dependent hadronic transport model IBUU04, we have studied the single and double neutron-proton differential transverse flow and its dependence on the nuclear symmetry energy in the semi-central reactions of $^{132}\text{Sn}+^{124}\text{Sn}$ and $^{112}\text{Sn}+^{112}\text{Sn}$ at beam energies of 400 and 800 MeV/nucleon. We find that the double neutron-proton differential flow retains about the same sensitivity to the symmetry energy as the single differential flow in the more neutron-rich system involved. Because

the double neutron-proton differential flow can reduce significantly both the systematic errors and effects of the Coulomb force, it is thus more useful probe for the high-density behavior of the nuclear symmetry energy.

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